

Monetary Economics

Extension 2: The Small-Open-Economy Extension

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Goal of the extension

- This extension immerses the basic NK model into an **open-economy setup** (nesting the closed economy as a special case).
- It introduces explicitly **key open-economy variables and concepts** such as
 - the exchange rate,
 - the terms of trade,
 - exports and imports,
 - international financial markets.
- It derives some important **positive and normative implications** of the openness of the economy for MP.

Galí and Monacelli's (2005) model I

- We consider Galí and Monacelli's (2005) model, which is a model
 - of a **small open economy**, not affecting the rest of the world,
 - with **no international-trade cost**, so that the law of one price holds,
 - with **complete international financial markets**, allowing for consumption-risk sharing across countries.

- For simplicity, this model abstracts from
 - non-tradable goods,
 - nominal-wage stickiness,
 - cost-push shocks.

Galí and Monacelli's (2005) model II

- In this model, the world economy is made of a **continuum** of infinitesimally small open economies represented by the unit interval $[0, 1]$.
- All these economies have the same **preferences, technology, and market structure**.
- The only shocks considered are **technology shocks**, which are imperfectly correlated across national economies.
- We consider a given small open economy, called the “**domestic economy**,” and we use the following notations:
 - variables without an i subscript refer to the domestic economy,
 - variables with an i subscript refer to the foreign economy $i \in [0, 1]$,
 - variables with an asterisk superscript ($*$) refer to the world economy.

Main results

- 1 There are two key equilibrium conditions, a **Phillips curve** and an **IS equation**, which are similar to their closed-economy counterparts.
- 2 There are **three sources of inefficiency**:
 - monopolistic competition,
 - price stickiness,
 - a terms-of-trade externality.
- 3 In a specific case, MP should have **two objectives**: stabilizing the output gap and “domestic inflation” (i.e. inflation in the price index for domestically produced goods).
- 4 In that specific case, **optimal MP** fully stabilizes domestic inflation.

Outline

- 1 Introduction
- 2 Households I
- 3 Households II
- 4 Firms
- 5 Equilibrium
- 6 Distortions
- 7 Loss function

Utility function

- The representative household (RH) of the domestic economy maximizes

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t U [C_t, N_t] \right\},$$

where U is the instantaneous utility function, identical to Chapter 1's, N_t is work hours, and C_t is a **composite consumption index** defined by

$$C_t \equiv \left[(1 - \alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}},$$

where

- $C_{H,t}$ is an index of dom. consumption of dom. goods (H for *Home*),
- $C_{F,t}$ is an index of dom. consumption of foreign goods (F for *Foreign*),
- $\alpha \in [0, 1]$ is a measure of **openness** (the case $\alpha = 0$ makes the model coincide with the closed-economy model studied in Chapters 1 to 3),
- $1 - \alpha$ is a measure of the degree of **home bias** in consumption,
- $\eta > 0$ is the dom. elasticity of subst. between dom. and foreign goods.

Consumption indexes I

- The index of **domestic consumption of domestic goods** is defined as

$$C_{H,t} \equiv \left[\int_0^1 C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where

- $C_{H,t}(j)$ denotes domestic consumption of domestic good j ,
 - $\varepsilon > 1$ is the elasticity of substitution between domestic goods.
- The index of **domestic consumption of foreign goods** is defined as

$$C_{F,t} \equiv \left[\int_0^1 (C_{i,t})^{\frac{\gamma-1}{\gamma}} di \right]^{\frac{\gamma}{\gamma-1}},$$

where

- $C_{i,t}$ is an index of dom. consumption of goods produced in country i ,
- $\gamma > 1$ is the elasticity of substitution between goods produced in different countries.

Consumption indexes II

- The fact that the definition of $C_{F,t}$ involves an integral over the continuum $[0, 1]$, which includes the domestic economy, does not matter since the latter has a zero measure.
- The index of **domestic consumption of goods produced in country i** is defined in the same way as $C_{H,t}$:

$$C_{i,t} \equiv \left[\int_0^1 C_{i,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where

- $C_{i,t}(j)$ denotes domestic consumption of good j produced in country i ,
- ε is also the elasticity of subst. between goods produced in country i .

Budget constraints

- RH faces the sequence of **budget constraints**

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj + \int_0^1 \int_0^1 P_{i,t}(j) C_{i,t}(j) dj di + \mathbb{E}_t \{ Q_{t,t+1} D_{t+1} \} \leq D_t + W_t N_t + T_t$$

for $t \geq 0$, where

- W_t is the nominal wage at date t ,
- T_t is lump-sum transfers (or minus lump-sum taxes) at date t ,
- $P_{H,t}(j)$ is the price of domestic good j at date t ,
- $P_{i,t}(j)$ is the price of good j imported from country i at date t ,
- D_{t+1} is the (random) nominal payoff at date $t+1$ of the portfolio of securities bought by RH at date t ,

all of them expressed in units of domestic currency, and

- $Q_{t,t+1}$ is the stochastic discount factor for one-period-ahead nominal payoffs relevant to RH at date t .

Distribution of consumption across goods I

- The optimized distribution of consumption across goods is characterized by five **demand schedules**, the first three of which are

$$C_{H,t}(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\varepsilon} C_{H,t}, \quad C_{i,t}(j) = \left[\frac{P_{i,t}(j)}{P_{i,t}} \right]^{-\varepsilon} C_{i,t},$$
$$C_{i,t} = \left[\frac{P_{i,t}}{P_{F,t}} \right]^{-\gamma} C_{F,t},$$

for all $(i, j) \in [0, 1]^2$ and $t \geq 0$, where, at each date $t \geq 0$,

- $P_{H,t} \equiv \left[\int_0^1 P_{H,t}(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}$ is an index of prices of domestic goods,
- $P_{i,t} \equiv \left[\int_0^1 P_{i,t}(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}$ is an index of prices of country i 's goods,
- $P_{F,t} \equiv \left[\int_0^1 P_{i,t}^{1-\gamma} di \right]^{\frac{1}{1-\gamma}}$ is an index of prices of imported goods,

all of them expressed in units of domestic currency.

Distribution of consumption across goods II

- The last two **demand schedules** are

$$C_{H,t} = (1 - \alpha) \left[\frac{P_{H,t}}{P_t} \right]^{-\eta} C_t,$$
$$C_{F,t} = \alpha \left[\frac{P_{F,t}}{P_t} \right]^{-\eta} C_t,$$

for all $t \geq 0$, where, at each date $t \geq 0$,

$$P_t \equiv \left[(1 - \alpha) (P_{H,t})^{1-\eta} + \alpha (P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}}$$

is the **consumer price index (CPI)**, expressed in units of domestic currency.

- When $\eta \rightarrow 1$ or (as will be the case at the steady state) $P_{H,t} = P_{F,t}$, parameter α corresponds to the share of domestic consumption allocated to imported goods, and represents therefore a **natural measure of openness**.

Rewriting the budget constraints

- Combining the demand schedules with the definitions of price and consumption indexes, we get, in the same way as in Chapter 1 and Extension 1,

$$\int_0^1 P_{H,t}(j) C_{H,t}(j) dj = P_{H,t} C_{H,t},$$

$$\int_0^1 P_{i,t}(j) C_{i,t}(j) dj = P_{i,t} C_{i,t},$$

$$\int_0^1 P_{i,t} C_{i,t} di = P_{F,t} C_{F,t},$$

$$P_{H,t} C_{H,t} + P_{F,t} C_{F,t} = P_t C_t,$$

for $i \in [0, 1]$ and $t \geq 0$, so that the date- t **budget constraint** can be rewritten as

$$P_t C_t + \mathbb{E}_t \{ Q_{t,t+1} D_{t+1} \} \leq D_t + W_t N_t + T_t.$$

Other intratemporal FOC of RH's optimization problem

- The other **intratemporal FOC** of RH's optimization problem is, as in Chapter 1,

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}.$$

- As in Chapter 1, given that $U(C_t, N_t) \equiv \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$, it can be rewritten as

$$\frac{W_t}{P_t} = C_t^\sigma N_t^\varphi,$$

or, in log-linearized form,

$$w_t - p_t = \sigma c_t + \varphi n_t.$$

Intertemporal FOC of RH's optimization problem I

- Consider a given **Arrow security**, i.e. a one-period security that yields one unit of domestic currency if a specific state of nature is realized at date $t + 1$ and nothing otherwise.
- The **intertemporal FOC** of RH's optimization problem can be written

$$\frac{V_{t,t+1} C_t^{-\sigma}}{P_t} = \frac{\tilde{\zeta}_{t,t+1} \beta C_{t+1}^{-\sigma}}{P_{t+1}},$$

where

- $V_{t,t+1}$ is the date- t price (in domestic currency) of this Arrow security,
- $\tilde{\zeta}_{t,t+1}$ is the probability that this state of nature is realized at date $t + 1$, conditional on the state of nature at date t ,
- C_{t+1} and P_{t+1} are here the values taken by the consumption index and the CPI at date $t + 1$ when this state of nature is realized.

Intertemporal FOC of RH's optimization problem II

- This FOC says that RH should be **indifferent** between purchasing one marginal unit of this Arrow security at date t or not:
 - the left-hand side is the utility **loss** resulting from the marginal decrease in C_t implied by this purchase,
 - the right-hand side is the utility **gain** resulting from the marg. increase in C_{t+1} in the corresponding state of nature implied by this purchase.
- The date- t price of a portfolio yielding a random payoff D_{t+1} at date $t + 1$ is

$$\sum_{\text{date-(}t+1\text{) states}} V_{t,t+1} D_{t+1} = \mathbb{E}_t \left\{ \frac{V_{t,t+1}}{\zeta_{t,t+1}} D_{t+1} \right\},$$

so that the **one-period stochastic discount factor** can be defined as

$$Q_{t,t+1} \equiv \frac{V_{t,t+1}}{\zeta_{t,t+1}}.$$

Intertemporal FOC of RH's optimization problem III

- Using this definition of $Q_{t,t+1}$, the previous FOC can be rewritten as

$$Q_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right)$$

for all possible states of nature at dates t and $t + 1$, which implies the same **Euler equation** as in Chapter 1 and Extension 1:

$$Q_t = \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\},$$

where $Q_t \equiv \mathbb{E}_t \{ Q_{t,t+1} \}$ is the date- t price of a one-period bond paying off one unit of domestic currency at date $t + 1$, so that the first-order approximation of this Euler equation around the ZIRSS can again be written as

$$c_t = \mathbb{E}_t \{ c_{t+1} \} - \frac{1}{\sigma} (i_t - \mathbb{E}_t \{ \pi_{t+1} \} - \bar{i}),$$

where $i_t \equiv -\log Q_t$ is the short-term nominal interest rate, $\bar{i} \equiv -\log \beta$ is the time-discount rate, and $\pi_t \equiv p_t - p_{t-1}$ is the **CPI inflation rate**.

Bilateral and effective terms of trade

- The **bilateral terms of trade** between the domestic economy and country i are defined as the price of country i 's goods in terms of home goods:

$$S_{i,t} \equiv \frac{P_{i,t}}{P_{H,t}}.$$

- The **effective terms of trade** are defined and obtained as

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}.$$

- Around a symmetric steady state with $S_{i,t} = 1$ for all $i \in [0, 1]$, they can be approximated, up to first order, by

$$s_t = \int_0^1 s_{i,t} di,$$

where $s_t \equiv \log S_t = p_{F,t} - p_{H,t}$.

Domestic and CPI inflation

- Around this symmetric steady state, the CPI definition can be approximated, up to first order, by

$$p_t = (1 - \alpha)p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha s_t,$$

- Define **domestic inflation** as the rate of change in the index of domestic-goods prices:

$$\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}.$$

- Domestic inflation and **CPI inflation** are then linked by

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t,$$

i.e. the gap between the two measures of inflation is proportional to the change in the effective terms of trade, the coefficient of proportionality being given by the measure of openness α .

Law of one price and bilateral nominal exchange rate

- In the absence of international-trade cost, the **law of one price** holds:

$$P_{i,t}(j) = \varepsilon_{i,t} P_{i,t}^i(j)$$

for all $(i, j) \in [0, 1]^2$, where

- $\varepsilon_{i,t}$ is the **bilateral nominal exchange rate** with country i (i.e. the price of country i 's currency in terms of the domestic currency),
 - $P_{i,t}^i(j)$ is the price of country i 's good j expressed in its own currency.
- The law of one price and the definition of $P_{i,t}$ together imply that

$$P_{i,t} = \varepsilon_{i,t} P_{i,t}^i$$

for all $i \in [0, 1]$, where $P_{i,t}^i \equiv \left[\int_0^1 P_{i,t}^i(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}}$ is country i 's domestic-price index expressed in its own currency.

Effective nominal exchange rate and world-price index

- Using the previous result to replace $P_{i,t}$ in the definition of $P_{F,t}$, we get, up to first order, around the symmetric steady state,

$$p_{F,t} = \int_0^1 (e_{i,t} + p_{i,t}^i) di = e_t + p_t^*,$$

where

- $e_{i,t} \equiv \log \mathcal{E}_{i,t}$,
 - $e_t \equiv \int_0^1 e_{i,t} di$ is the (log) **effective nominal exchange rate**,
 - $p_t^* \equiv \int_0^1 p_{i,t}^i di$ is the (log) **world-price index** (for the world as a whole, there is no distinction between the CPI and the domestic-price index).
- Therefore, the effective terms of trade can be written as

$$s_t = e_t + p_t^* - p_{H,t}.$$

Bilateral and effective real exchange rates

- Define the **bilateral real exchange rate** with country i as the ratio of the two countries' CPIs, both expressed in terms of domestic currency:

$$Q_{i,t} \equiv \frac{\varepsilon_{i,t} P_t^i}{P_t},$$

where P_t^i denotes country i 's CPI expressed in terms of country i 's currency.

- Define the (log) **effective real exchange rate** as $q_t \equiv \int_0^1 q_{i,t} di$, where $q_{i,t} \equiv \log Q_{i,t}$.
- We then have, up to first order,

$$q_t = \int_0^1 (e_{i,t} + p_t^i - p_t) di = e_t + p_t^* - p_t = s_t + p_{H,t} - p_t = (1 - \alpha)s_t.$$

International risk sharing I

- Given that the Arrow securities are traded internationally, the **intertemporal FOC** of the optimization problem of any country i 's RH can be written as

$$\frac{V_{t,t+1} (C_t^i)^{-\sigma}}{\varepsilon_{i,t} P_t^i} = \frac{\tilde{\zeta}_{t,t+1} \beta (C_{t+1}^i)^{-\sigma}}{\varepsilon_{i,t+1} P_{t+1}^i}$$

for any Arrow security whose price ($V_{t,t+1}$) and payoff (equal to 1) are expressed in domestic currency, where C_{t+1}^i and P_{t+1}^i are conditional on the state of nature corresponding to the Arrow security considered.

- In the same way as previously, this FOC is shown to imply

$$Q_{t,t+1} = \beta \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \left(\frac{P_t^i}{P_{t+1}^i} \right) \left(\frac{\varepsilon_{i,t}}{\varepsilon_{i,t+1}} \right)$$

for all states of nature, all $i \in [0, 1]$, and all $t \geq 0$.

International risk sharing II

- This equation and its domestic counterpart together imply

$$\frac{C_{t+1}}{C_t} = \left(\frac{C_{t+1}^i}{C_t^i} \right) \left(\frac{Q_{i,t+1}}{Q_{i,t}} \right)^{\frac{1}{\sigma}}$$

for all states of nature, all $i \in [0, 1]$, and all $t \geq 0$, which in turn implies

$$C_t = \vartheta_i C_t^i Q_{i,t}^{\frac{1}{\sigma}}$$

for all states of nature, all $i \in [0, 1]$, and all $t \geq 0$, where ϑ_i is a constant depending on initial net foreign asset positions.

- We assume zero initial net foreign asset positions, so that $\vartheta_i = 1$ for all $i \in [0, 1]$ and the previous condition becomes, in aggregate log terms,

$$c_t = c_t^* + \frac{1}{\sigma} q_t \simeq c_t^* + \left(\frac{1 - \alpha}{\sigma} \right) s_t,$$

where $c_t^* \equiv \int_0^1 c_t^i di$ denotes the (log) **world-consumption index**.

Uncovered interest-rate parity

- The domestic-currency price of a riskless bond denominated in country i 's currency is

$$\varepsilon_{i,t} Q_t^i = \mathbb{E}_t \{ \varepsilon_{i,t+1} Q_{t,t+1} \},$$

where Q_t^i is the price of the bond in country i 's currency.

- This pricing equation, combined with the domestic-bond-pricing equation $Q_t = \mathbb{E}_t \{ Q_{t,t+1} \}$ and the definition $i_t^i \equiv -\log(Q_t^i)$, implies

$$\mathbb{E}_t \left\{ Q_{t,t+1} \left[\exp(i_t) - \frac{\varepsilon_{i,t+1}}{\varepsilon_{i,t}} \exp(i_t^i) \right] \right\} = 0.$$

- The latter condition, approximated around the steady state and aggregated over $i \in [0, 1]$, gives the **uncovered interest-rate parity**

$$i_t = i_t^* + \mathbb{E}_t \{ \Delta e_{t+1} \}.$$

Technology

- In this extension, for simplicity, we restrict the analysis to a **linear technology**:

$$Y_t(j) = A_t N_t(j),$$

where $j \in [0, 1]$ indexes the continuum of firms.

- Therefore, the **real marginal cost** (expressed in domestic goods) is common across domestic firms and given by

$$mc_t = -\nu + w_t - p_{H,t} - a_t,$$

where $\nu \equiv -\log(1 - \tau)$, with τ being the employment subsidy.

Price setting

- As in Chapter 1, we assume that, at each date,
 - only a fraction $1 - \theta$ of firms, drawn randomly from the population, are allowed to reset their price, where $0 \leq \theta \leq 1$,
 - an individual firm's probability of being allowed to reset its price is independent of the time elapsed since it last reset its price.
- As shown in Chapter 1, the newly reset (log) domestic price at date t , noted $\bar{p}_{H,t}$, can be approximated as

$$\bar{p}_{H,t} = \mu + (1 - \beta\theta) \sum_{k=0}^{+\infty} (\beta\theta)^k \mathbb{E}_t \{ mc_{t+k} + p_{H,t+k} \},$$

where $\mu \equiv \log\left(\frac{\varepsilon}{\varepsilon-1}\right)$ is the (log) gross steady-state markup, or, equivalently, the (log) gross flexible-price markup.

Goods-market-clearing conditions I

- The **domestic-goods-market-clearing conditions** are

$$Y_t(j) = C_{H,t}(j) + \int_0^1 C_{H,t}^i(j) di$$

for all $j \in [0, 1]$ and $t \geq 0$, where $C_{H,t}^i(j)$ denotes country i 's demand for domestic good j .

- Using the domestic demand schedules and the assumption of symmetric preferences across countries, we get

$$C_{H,t}^i(j) = \alpha \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\varepsilon} \left(\frac{P_{H,t}}{\varepsilon_{i,t} P_{F,t}^i} \right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^i.$$

Goods-market-clearing conditions II

- Therefore, the goods-market-clearing conditions can be rewritten as

$$Y_t(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\varepsilon} \left[(1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\varepsilon_{i,t} P_{F,t}^i} \right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^i di \right].$$

- Plugging this expression for $Y_t(j)$ into $Y_t \equiv \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$ yields

$$\begin{aligned} Y_t &= (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \alpha \int_0^1 \left(\frac{P_{H,t}}{\varepsilon_{i,t} P_{F,t}^i} \right)^{-\gamma} \left(\frac{P_{F,t}^i}{P_t^i} \right)^{-\eta} C_t^i di \\ &= \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} \left[(1-\alpha) C_t + \alpha \int_0^1 \left(\frac{\varepsilon_{i,t} P_{F,t}^i}{P_{H,t}} \right)^{\gamma-\eta} Q_{i,t}^\eta C_t^i di \right]. \end{aligned}$$

Goods-market-clearing conditions III

- Using $C_t = C_t^i Q_{i,t}^{\frac{1}{\sigma}}$, we can rewrite the previous condition as

$$Y_t = \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \left[(1 - \alpha) + \alpha \int_0^1 \left(S_t^i S_{i,t} \right)^{\gamma - \eta} Q_{i,t}^{\eta - \frac{1}{\sigma}} di \right],$$

where

- $S_t^i \equiv \frac{\varepsilon_{i,t} P_{F,t}^i}{P_{i,t}}$ is the effective terms of trade of country i ,
 - $S_{i,t} \equiv \frac{P_{i,t}}{P_{H,t}}$ is the bilateral terms of trade with country i .
- Using $\int_0^1 s_t^i di = 0$, we can approximate this condition around the symmetric steady state as

$$y_t = c_t + \alpha \gamma s_t + \alpha \left(\eta - \frac{1}{\sigma} \right) q_t = c_t + \frac{\alpha \omega}{\sigma} s_t,$$

where $\omega \equiv \sigma \gamma + (1 - \alpha)(\sigma \eta - 1)$.

Goods-market-clearing conditions IV

- A similar condition holds for any country $i \in [0, 1]$:

$$y_t^i = c_t^i + \frac{\alpha\omega}{\sigma} s_t^i.$$

- By aggregating over countries $i \in [0, 1]$ and using again $\int_0^1 s_t^i di = 0$, we get the **world goods-market-clearing condition**

$$y_t^* \equiv \int_0^1 y_t^i di = \int_0^1 c_t^i di \equiv c_t^*,$$

where y_t^* and c_t^* are (log) indexes for world output and consumption.

- Using this condition, $c_t = c_t^* + \left(\frac{1-\alpha}{\sigma}\right) s_t$, and $y_t = c_t + \frac{\alpha\omega}{\sigma} s_t$, we get

$$y_t = y_t^* + \frac{1}{\sigma_\alpha} s_t,$$

where $\sigma_\alpha \equiv \frac{\sigma}{1+\alpha(\omega-1)} > 0$.

Rewriting the Euler equation

- Using sequentially $y_t = c_t + \frac{\alpha\omega}{\sigma}s_t$, $\pi_t = \pi_{H,t} + \alpha\Delta s_t$, and $y_t = y_t^* + \frac{1}{\sigma_\alpha}s_t$, we can rewrite the **Euler equation** as

$$\begin{aligned} y_t &= \mathbb{E}_t \{y_{t+1}\} - \frac{1}{\sigma} (i_t - \mathbb{E}_t \{\pi_{t+1}\} - \bar{i}) - \frac{\alpha\omega}{\sigma} \mathbb{E}_t \{\Delta s_{t+1}\} \\ &= \mathbb{E}_t \{y_{t+1}\} - \frac{1}{\sigma} (i_t - \mathbb{E}_t \{\pi_{H,t+1}\} - \bar{i}) - \frac{\alpha^\Theta}{\sigma} \mathbb{E}_t \{\Delta s_{t+1}\} \\ &= \mathbb{E}_t \{y_{t+1}\} - \frac{1}{\sigma_\alpha} (i_t - \mathbb{E}_t \{\pi_{H,t+1}\} - \bar{i}) + \alpha^\Theta \mathbb{E}_t \{\Delta y_{t+1}^*\}, \end{aligned}$$

where $\Theta \equiv \omega - 1$.

- Thus, when $\Theta > 0$ (i.e. for relatively large values of η and γ), an increase in the degree of openness (α) raises the sensitivity ($\frac{1}{\sigma_\alpha}$) of domestic output to the domestic real interest rate ($i_t - \mathbb{E}_t \{\pi_{H,t+1}\}$), given world output.
- It does so by amplifying the **real appreciation** (and the consequent switch of expenditures towards foreign goods) induced by a given interest-rate rise.

Trade balance

- Let $nx_t \equiv \frac{1}{Y} \left(Y_t - \frac{P_t}{P_{H,t}} C_t \right)$ denote **net exports** in terms of domestic output, expressed as a fraction of steady-state output Y .
- We get, at the first order,

$$nx_t = y_t - c_t - \alpha s_t.$$

- Together with $y_t = c_t + \frac{\alpha\omega}{\sigma} s_t$, this implies

$$nx_t = \alpha \left(\frac{\omega}{\sigma} - 1 \right) s_t.$$

- Therefore, the relationship between net exports and the terms of trade may be positive or negative, depending on the values of the structural parameters.

Aggregate production function

- Using the **individual production function** $Y_t(j) = A_t N_t(j)$, we get

$$N_t \equiv \int_0^1 N_t(j) dj = \frac{1}{A_t} \int_0^1 Y_t(j) dj = \frac{Y_t}{A_t} \int_0^1 \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\varepsilon} dj.$$

- Lemma 1 (established in Chapter 2) implies that variations in $d_t \equiv \int_0^1 \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\varepsilon} dj$ around the steady state are of second order.

- We therefore get, at the first order, the following **aggregate production function**:

$$y_t = a_t + n_t.$$

Domestic inflation and marginal cost

- As in Chapter 1, the equation describing the dynamics of the domestic-goods-price index as a function of newly set domestic prices,

$$\pi_{H,t} = (1 - \theta)(\bar{p}_{H,t} - p_{H,t-1}),$$

can be combined with the FOC of firms' optimization problem to yield

$$\pi_{H,t} = \beta \mathbb{E}_t \{ \pi_{H,t+1} \} + \chi \widehat{m\hat{c}}_t,$$

where $\chi \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$.

- Thus, the relationship between domestic inflation and the domestic marginal cost does not depend on any open-economy parameter.

Marginal cost I

- Using $w_t - p_t = \sigma c_t + \varphi n_t$, $p_t - p_{H,t} = \alpha s_t$, $c_t = c_t^* + \left(\frac{1-\alpha}{\sigma}\right) s_t$, $c_t^* = y_t^*$, and $y_t = a_t + n_t$, we can express the **domestic marginal cost** mc_t as

$$\begin{aligned}
 mc_t &= -v + (w_t - p_{H,t}) - a_t \\
 &= -v + (w_t - p_t) + (p_t - p_{H,t}) - a_t \\
 &= -v + \sigma c_t + \varphi n_t + \alpha s_t - a_t \\
 &= -v + \sigma y_t^* + \varphi y_t + s_t - (1 + \varphi) a_t.
 \end{aligned}$$

- So the domestic marginal cost mc_t depends **positively on domestic output** y_t , through its effect on employment n_t and, hence, the real wage $w_t - p_t$ (because of convex labor disutility: $\varphi > 0$).
- It depends **negatively on technology** a_t , through
 - its direct effect on labor productivity,
 - its effect on n_t and, hence, $w_t - p_t$, for a given y_t .

Marginal cost II

- It depends **positively on world output** y_t^* , through its effect on domestic consumption c_t (via international risk sharing) and, hence, the real wage $w_t - p_t$ (because of concave consumption utility: $\sigma > 0$).
- Lastly, it depends **positively on the terms of trade** s_t , through
 - their effect on c_t and, hence, $w_t - p_t$, for a given y_t^* ,
 - their direct effect on $w_t - p_{H,t}$ for a given $w_t - p_t$.

- Using $y_t = y_t^* + \frac{1}{\sigma_\alpha} s_t$, we can rewrite mc_t as

$$mc_t = -v + (\sigma_\alpha + \varphi) y_t + (\sigma - \sigma_\alpha) y_t^* - (1 + \varphi) a_t.$$

- So domestic output y_t affects the domestic marginal cost mc_t through
 - its effect on employment (captured by φ),
 - its effect on the terms of trade (captured by σ_α).

Marginal cost III

- World output y_t^* affects the domestic marginal cost mc_t through
 - its effect on consumption (captured by σ),
 - its effect on the terms of trade (captured by σ_α).
- When $\Theta > 0$ (i.e. for relatively high substitutability between goods produced in different countries), we have $\sigma > \sigma_\alpha$, so that the domestic marginal cost mc_t depends **positively on world output** y_t^* .
- The reason is that the size of the real appreciation needed to absorb the change in relative supplies is then relatively small.
- When $\Theta > 0$, an increase in openness α
 - decreases the sensitivity of mc_t (and hence $\pi_{H,t}$) to y_t ,
 - increases the sensitivity of mc_t (and hence $\pi_{H,t}$) to y_t^* ,
 by reducing the size of the required adjustment in the terms of trade.

Natural level of output and output gap

- Define the **natural level of output** y_t^n as the level of domestic output that would prevail if prices were flexible in the domestic economy and sticky elsewhere.
- Since the firms' FOC implies that $mc_t = -\mu$ under flexible prices, we get

$$y_t^n = \Gamma_0 + \Gamma_a a_t + \Gamma_* y_t^*,$$

where $\Gamma_0 \equiv \frac{\nu - \mu}{\sigma_\alpha + \varphi}$, $\Gamma_a \equiv \frac{1 + \varphi}{\sigma_\alpha + \varphi}$, and $\Gamma_* \equiv -\frac{\alpha \Theta \sigma_\alpha}{\sigma_\alpha + \varphi}$ (≤ 0 depending on the relative importance of the terms-of-trade effect discussed above).

- Using the last expressions for mc_t and y_t^n , we get

$$\widehat{mc}_t = (\sigma_\alpha + \varphi) \widetilde{y}_t,$$

where $\widetilde{y}_t \equiv y_t - y_t^n$ is the **output gap**.

Phillips curve

- Using the last expression to replace \widehat{mc}_t in the firms's FOC, we get the following **Phillips curve**:

$$\pi_{H,t} = \beta \mathbb{E}_t \{ \pi_{H,t+1} \} + \kappa_\alpha \tilde{y}_t,$$

where $\kappa_\alpha \equiv (\sigma_\alpha + \varphi) \chi$.

- This small-open-economy Phillips curve is **isomorphic** to its closed-economy counterpart.
- The **main difference** is that the degree of openness α affects the slope κ_α of the small-open-economy Phillips curve.
- More precisely, when $\Theta > 0$, an increase in α decreases κ_α , by reducing the real depreciation induced by an increase in domestic output and, hence, the effect of domestic output on marginal cost and inflation.

IS equation

- Using the expression for y_t^n to rewrite the Euler equation, we get the following **IS equation**:

$$\tilde{y}_t = \mathbb{E}_t \{ \tilde{y}_{t+1} \} - \frac{1}{\sigma_\alpha} (i_t - \mathbb{E}_t \{ \pi_{H,t+1} \} - r_t^n),$$

where $r_t^n \equiv \bar{i} - \sigma_\alpha \Gamma_\alpha (1 - \rho_a) a_t + \frac{\alpha \ominus \sigma_\alpha \varphi}{\sigma_\alpha + \varphi} \mathbb{E}_t \{ \Delta y_{t+1}^* \}$ is the domestic **natural rate of interest**.

- This small-open-economy IS equation is **isomorphic** to its closed-economy counterpart.
- The **main differences** are that, in the small-open-economy IS equation,
 - the degree of openness α influences the sensitivity $\frac{1}{\sigma_\alpha}$ of the output gap to interest-rate changes,
 - the natural rate of interest r_t^n depends on expected world-output growth $\mathbb{E}_t \{ \Delta y_{t+1}^* \}$, in addition to domestic productivity a_t .

Taylor principle

- Given $(a_t, i_t)_{t \in \mathbb{N}}$, $(\tilde{y}_t, \pi_{H,t})_{t \in \mathbb{N}}$ is determined by
 - the IS equation $\tilde{y}_t = \mathbb{E}_t \{ \tilde{y}_{t+1} \} - \frac{1}{\sigma_\alpha} (i_t - \mathbb{E}_t \{ \pi_{H,t+1} \} - r_t^n)$,
 - the Phillips curve $\pi_{H,t} = \beta \mathbb{E}_t \{ \pi_{H,t+1} \} + \kappa_\alpha \tilde{y}_t$,

for $t \in \mathbb{N}$.

- Given the isomorphism between the closed- and small-open-economy Phillips curve and IS equation, we get the same **determinacy conditions** as in Chapter 3 for the same parametric families of rules, except that
 - σ and κ should be replaced by σ_α and κ_α in the conditions,
 - π and x should be replaced by π_H and \tilde{y} in the rules.
- Therefore, we get the same **Taylor principle** as in Chapter 3: in the long term, the (nominal) interest rate should rise by more than the increase in the domestic inflation rate in order to ensure determinacy.

A special case

- In the rest of the extension (devoted to normative issues), we focus on the **special case** in which $\sigma = \eta = \gamma = 1$.
- In this special case, the following equilibrium conditions, previously obtained as first-order approximations, hold **exactly**:

$$s_t = \int_0^1 s_{i,t} di, \quad p_t = p_{H,t} + \alpha s_t, \quad \pi_t = \pi_{H,t} + \alpha \Delta s_t,$$

$$q_t = (1 - \alpha) s_t, \quad c_t = c_t^* + \left(\frac{1 - \alpha}{\sigma} \right) s_t = c_t^* + (1 - \alpha) s_t,$$

$$y_t = c_t + \frac{\alpha \omega}{\sigma} s_t = c_t + \alpha s_t, \quad n x_t = 0.$$

- Moreover, we have $\omega = 1$, $\sigma_\alpha = \sigma$, $\Theta = 0$, $\Gamma_* = 0$, and $\kappa_\alpha = \kappa$, so that the small-open-economy IS equation and Phillips curve are exactly **identical** to their closed-economy counterparts.

Social-planner allocation I

- Consider a **benevolent social planner**, seeking to maximize the welfare of the domestic economy's RH, subject to
 - the technology constraint,
 - the same resource constraints as those faced by the domestic economy vis-à-vis the rest of the world, given the complete-markets assumption.
- Given the absence of state variable (such as the capital stock), its optimization problem is **static**: at each date t , taking C_t^* as given,

$$\underset{C_t, N_t}{\text{Max}} U(C_t, N_t)$$

subject to

- the tech. constraint $Y_t = A_t N_t$ (output being the same across goods),
- the international-risk-sharing condition $C_t = C_t^* S_t^{1-\alpha}$,
- the goods-market-clearing condition $Y_t = C_t S_t^\alpha$.

Social-planner allocation II

- These three constraints, together with the world goods-market-clearing condition $C_t^* = Y_t^*$, can be summarized by

$$C_t = A_t^{1-\alpha} (Y_t^*)^\alpha N_t^{1-\alpha}.$$

- The **optimality condition** equalizes the MRS between consumption and work to the corresponding marginal rate of transformation:

$$-\frac{U_{n,t}}{U_{c,t}} = (1 - \alpha) \frac{C_t}{N_t},$$

which implies

$$N_t = (1 - \alpha)^{\frac{1}{1+\varphi}} \quad \text{and} \quad C_t = (1 - \alpha)^{\frac{1-\alpha}{1+\varphi}} A_t^{1-\alpha} (Y_t^*)^\alpha.$$

- Thus, at the **social-planner allocation**,
 - employment N_t is constant over time,
 - cons. C_t fluctuates in response to technology A_t and world output Y_t^* .

Distortions

- The model is characterized by **three distortions**:
 - 1 monopolistic competition in the goods market,
 - 2 price stickiness,
 - 3 a terms-of-trade externality between countries.

- The **first two distortions** are the same as in Chapter 2 and Extension 1.

- As noted by Corsetti and Pesenti (2001) and Benigno and Benigno (2003), the **third distortion** comes from the CB's ability to influence the terms of trade in a way beneficial to domestic consumers, due to
 - the imperfect substitutability between domestic and foreign goods,
 - price stickiness, making MP not neutral.

Condition for natural-allocation efficiency I

- Define the **natural allocation** as the equilibrium allocation when prices are flexible in the domestic economy and sticky elsewhere.
- Since $\eta = 1$, we have $P_t = P_{H,t}^{1-\alpha} P_{F,t}^\alpha$ and hence $\frac{P_t}{P_{H,t}} = \left(\frac{P_{F,t}}{P_{H,t}}\right)^\alpha = \mathcal{S}_t^\alpha$.
- Using this result, RH's intratemporal FOC $-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$, the goods-market-clearing condition $Y_t = C_t \mathcal{S}_t^\alpha$, and the flexible-price aggregate production function $Y_t = A_t N_t$, we can characterize the natural allocation by

$$\begin{aligned} \frac{\varepsilon - 1}{\varepsilon} &= MC_t = \frac{(1 - \tau)W_t}{A_t P_{H,t}} = \frac{(1 - \tau)W_t \mathcal{S}_t^\alpha}{A_t P_t} = -\frac{(1 - \tau)\mathcal{S}_t^\alpha U_{n,t}}{A_t U_{c,t}} \\ &= -\frac{(1 - \tau)Y_t U_{n,t}}{A_t C_t U_{c,t}} = \frac{(1 - \tau)Y_t N_t^\varphi C_t}{A_t C_t} = (1 - \tau)N_t^{1+\varphi}. \end{aligned}$$

Condition for natural-allocation efficiency II

- Therefore, the value

$$\tau = 1 - \frac{\varepsilon - 1}{(1 - \alpha)\varepsilon}$$

is such that the corresponding **natural allocation is efficient** (i.e. coincides with the social-planner allocation).

- This value depends not only on the elasticity of substitution between goods ε , as in Chapter 2, but also on the degree of openness α .
- This is because openness creates a **terms-of-trade externality** between countries, which distorts the incentives of CB beyond monop. competition.
- I.e., an employment subsidy exactly offsetting the monopolistic-competition distortion would not make the natural allocation efficient, because CB would have an incentive to deviate from it in order to improve the terms of trade.

MP and the (efficient) natural allocation I

- In the rest of the extension, we assume that $\tau = 1 - \frac{\varepsilon-1}{(1-\alpha)\varepsilon}$, so that the natural allocation is efficient.
- As in Chapter 2, and as apparent from the IS equation and the Phillips curve, MP can then achieve the (efficient) natural allocation ($\tilde{y}_t = 0$) by
 - making the interest rate track the natural rate of interest: $i_t = r_t^n$,
 - **stabilizing domestic inflation** at a constant (zero) level: $\pi_{H,t} = 0$.
- As in Chapter 2, this is because the flexible-price allocation can be replicated when prices are sticky by making all firms satisfied with their existing prices, so that **the sticky-price constraint is not binding**.
- Because it perfectly stabilizes domestic inflation, this optimal MP is sometimes called “**(strict) domestic-inflation targeting**” (DIT).

MP and the (efficient) natural allocation II

- Under DIT, in response to a positive tech. shock, for given world variables,
 - domestic output increases: $y_t = \Gamma_0 + \Gamma_a a_t$ with $\Gamma_a > 0$,
 - the terms of trade increase, i.e. deteriorate: $s_t = \sigma(y_t - y_t^*)$,
 - the real exchange rate increases, i.e. depreciates: $q_t = (1 - \alpha)s_t$,
 - the nom. exch. rate increases, i.e. depreciates: $e_t = s_t - p_t^* + p_{H,t}$,
 - the CPI increases: $p_t = p_{H,t} + \alpha s_t$.
- Under DIT and constant world prices p_t^* , **the lower the correlation** between domestic natural output y_t^n (or, equivalently, domestic productivity a_t) and world output y_t^* , **the higher the volatility** of
 - the terms of trade s_t ,
 - the real exchange rate q_t ,
 - the nominal exchange rate e_t ,
 - the CPI p_t .

MP and the (efficient) natural allocation III

- Under DIT, for a given correlation between domestic natural output y_t^n and world output y_t^* , an increase in the degree of openness α
 - has no effect on the volatility of domestic output y_t ,
 - has no effect on the volatility of the terms of trade s_t ,
 - has no effect on the volatility of the nominal exchange rate e_t ,
 - decreases the volatility of the real exchange rate q_t ,
 - increases the volatility of the CPI p_t .
- Thus, optimal MP (i.e. DIT) may entail **large movements in the nominal exchange rate and CPI inflation**, especially for an economy that is very open and subject to largely idiosyncratic shocks.
- This is because optimal MP allows the nominal exchange rate and CPI inflation to adjust as needed in order to replicate the flexible-price response of the terms of trade, given that domestic prices are constant.

Motivation for determining the welfare-loss function

- The **efficient allocation** is feasible only if CB observes a_t or some date- t endogenous variables at each date t (so that $i_t = r_t^n$ in equilibrium).
- When this condition is not met, MP cannot achieve the efficient allocation, so that **optimal feasible MP** cannot be derived from the efficient allocation.
- In that case, optimal feasible MP is obtained by minimizing the **welfare-loss function**, i.e. the second-order approximation of RH's utility function, subject to the structural equations and CB's observation-set constraint.
- This welfare-loss function tells us
 - the **objectives** that MP should have,
 - the **weight** that CB should put on each objective.

Determination of the welfare-loss function I

- We now derive the second-order approximation of RH's **intertemporal utility** in the neighborhood of the symmetric steady state.
- The exact relationships $c_t = c_t^* + (1 - \alpha) s_t$ and $y_t = c_t + \alpha s_t$ imply that **instantaneous consumption utility** can be written exactly as

$$\log C_t = c_t = (1 - \alpha) y_t + \alpha c_t^* = (1 - \alpha) \hat{y}_t + t.i.p.,$$

where *t.i.p.* stands for “terms independent of policy.”

- As in Chapter 2 and Extension 1, note that, for any variable Z_t , we have

$$\frac{Z_t - Z}{Z} \simeq \hat{z}_t + \frac{\hat{z}_t^2}{2},$$

where $\hat{z}_t \equiv z_t - z$ is the log-deviation of Z_t from its steady-state value.

Determination of the welfare-loss function II

- Therefore, **instantaneous labor disutility** can be approximated, up to second order, as

$$\begin{aligned} \frac{N_t^{1+\varphi}}{1+\varphi} &\simeq \frac{N^{1+\varphi}}{1+\varphi} + N^{1+\varphi} \left(\frac{N_t - N}{N} \right) + \frac{\varphi N^{1+\varphi}}{2} \left(\frac{N_t - N}{N} \right)^2 \\ &\simeq \frac{N^{1+\varphi}}{1+\varphi} + N^{1+\varphi} \left(\hat{n}_t + \frac{1+\varphi}{2} \hat{n}_t^2 \right). \end{aligned}$$

- Now, recall that

$$\hat{y}_t = \hat{n}_t + a_t - d_t,$$

where $d_t \equiv \int_0^1 \left[\frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\varepsilon} dj$.

Determination of the welfare-loss function III

- Moreover, we know from Lemma 1 (established in Chapter 2) that, up to a second-order approximation,

$$d_t \simeq \frac{\varepsilon}{2} \text{var}_j \{p_{H,t}(j)\}.$$

- We can therefore rewrite **instant. labor disutility**, up to second order, as

$$\frac{N_t^{1+\varphi}}{1+\varphi} \simeq \frac{N^{1+\varphi}}{1+\varphi} + N^{1+\varphi} \left[\hat{y}_t + \frac{\varepsilon}{2} \text{var}_j \{p_{H,t}(j)\} + \frac{1+\varphi}{2} (\hat{y}_t - a_t)^2 \right] + t.i.p.$$

- Our optimal-employment-subsidy assumption, $\tau = 1 - \frac{\varepsilon-1}{(1-\alpha)\varepsilon}$, implies

$$N^{1+\varphi} = 1 - \alpha.$$

Determination of the welfare-loss function IV

- Therefore, a second-order approximation of **instantaneous utility** is

$$\begin{aligned} \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} &\simeq -(1-\alpha) \left[\frac{\varepsilon}{2} \text{var}_j \{p_{H,t}(j)\} + \frac{1+\varphi}{2} (\hat{y}_t - a_t)^2 \right] + t.i.p. \\ &\simeq -(1-\alpha) \left[\frac{\varepsilon}{2} \text{var}_j \{p_{H,t}(j)\} + \frac{1+\varphi}{2} (\tilde{y}_t)^2 \right] + t.i.p., \end{aligned}$$

where the second equality comes from the fact that, at the first order,

$$\hat{y}_t - a_t = y_t - y - a_t \simeq y_t - y_t^n = \tilde{y}_t.$$

- Finally, we know from Lemma 2 (stated in Chapter 2) that, up to a second-order approximation,

$$\sum_{t=0}^{+\infty} \beta^t \text{var}_j \{p_{H,t}(j)\} \simeq \frac{1}{\chi} \sum_{t=0}^{+\infty} \beta^t (\pi_{H,t})^2.$$

Determination of the welfare-loss function V

- Therefore, we get that, up to second order, $\mathbb{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t U_t \right\} \simeq$

$$- \left(\frac{1-\alpha}{2} \right) \mathbb{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t \left[\frac{\varepsilon}{\chi} (\pi_{H,t})^2 + (1+\varphi) (\tilde{y}_t)^2 \right] \right\} + t.i.p.$$

- Hence the **welfare-loss function**

$$L_0 \equiv \mathbb{E}_0 \left\{ \sum_{t=0}^{+\infty} \beta^t \left[(\pi_{H,t})^2 + \lambda (\tilde{y}_t)^2 \right] \right\},$$

where $\lambda \equiv \frac{(1+\varphi)\chi}{\varepsilon}$.

Determination of the welfare-loss function VI

- This welfare-loss function is **identical** to its closed-economy counterpart, obtained in Chapter 2, for
 - no steady-state distortion,
 - no cost-push shocks,
 - constant returns to scale,
 - an elasticity of intertemporal substitution equal to one,with dom. inflation, not CPI inflation, being the relevant inflation variable.

- It can be **interpreted** in exactly the same way as in Chapter 2.